Designing Markets for Daily Deals

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Abstract

Daily deals platforms such as Amazon Local, Google Offers, GroupOn, and LivingSocial have provided a new channel for merchants to directly market to consumers. In order to maximize consumer acquisition and retention, these platforms would like to offer deals that give good value to users. Currently, selecting such deals is done manually; however, the large number of submarkets and localities necessitates an automatic approach to selecting good deals and determining merchant payments.

We approach this challenge as a market design problem. We postulate that merchants already have a good idea of the attractiveness of their deal to consumers as well as the amount they are willing to pay to offer their deal. The goal is to design an auction that maximizes a combination of the revenue of the auctioneer (platform), welfare of the bidders (merchants), and the positive externality on a third party (the consumer), despite the asymmetry of information about this consumer benefit. We design auctions that truthfully elicit this information from the merchants and maximize the social welfare objective, and we characterize the consumer welfare functions for which this objective is truthfully implementable. We generalize this characterization to a very broad mechanism-design setting and give examples of other applications.

1 Introduction

Daily deals websites such as Amazon Local, Google Offers, GroupOn, and LivingSocial have provided a new channel of direct marketing for merchants.

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In contrast to standard models of advertising such as television ads and web search results, the daily deals setting provides two new challenges to platforms.

First, in models of advertising such as web search, the advertisement is shown on the side of the main content; in contrast, daily deals websites offer consumers web pages or emails that contain only advertisements (*i.e.*, coupons). Therefore, for the long-term success of a platform, the decision of which coupons to show to the user must depend heavily on the benefit these coupons provide to consumers.

Second, the merchant often has significantly more information than the advertising platform about this consumer benefit. This benefit depends on many things: how much discount the coupon is offering, how the undiscounted price compares with the price of similar goods at the competitors, the price elasticity of demand for the good, the fine prints of the coupon, and so on. These parameters are known to the merchants, who routinely use such information to optimize their pricing and their inventory, but not to the platform provider who cannot be expected to be familiar with all markets and would need to invest significant resources to learn these parameters. Furthermore, unlike standard advertising models where an ad is displayed over a time period to a number of users and its value to the user (often measured using proxies like click-through rate or conversion rate) can be estimated over time, the structure of the daily deals market does not permit much experimentation: A number of deals must be selected at the beginning of each day to be sent to the subscribers all at once, and the performance of previous coupons, if any, by the same advertiser is not a good predictor of the performance of the current coupon, as changing any of the terms of the coupons can significantly affect its value.

These challenges pose a novel *market design* problem: How can we select deals with good benefit to the consumer in the presence of strongly asymmetric information about this benefit? This is precisely our goal in this paper. We postulate that merchants hold, as private information, two parameters: A *valuation* equalling the overall utility the merchant gains from being selected (as in a standard auction); and a *quality* that represents the attractiveness of their deal to a user. The task is to design an auction mechanism that incentivizes the merchants to reveal their private information about both their valuation and quality, then picks deals that maximize a combination of platform, merchant, and consumer values. We show that, if consumer welfare is a convex function of quality, then we can design a truthful auction that maximizes total social welfare; furthermore, we show that the convexity condition is necessary. We give negative results for another natural goal, achieving a constant-fraction welfare objective subject to a quality threshold guarantee. The main idea behind our positive results is to design a mechanism where bidders' total payment is contingent (in a carefully chosen way) upon whether the consumer purchases the coupon. Not surprisingly, the theory of proper scoring rules comes in handy here.

We then extend these results to characterize incentive-compatible mechanisms for social welfare maximization in a very general auction setting, where the type of each bidder has both a valuation and a quality component. Quality is modeled as a distribution over possible states of the world; a *consumer welfare function* maps these distributions to the welfare of some non-bidding party. We design truthful welfare-maximizing mechanisms for this setting and characterize implementable consumer welfare functions with a convexity condition that captures expected welfare and, intuitively, risk-averse preferences. We give a number of example applications demonstrating that our framework can be applied in a broad range of mechanism design settings, from network design to principal agent problems.

The rest of this paper is organized as follows: In the next section, we formally define the setting and the problem. In Section 3, we give a mechanism for maximizing social welfare when consumer welfare is a convex function the quality. In Section 4, we show that no truthful mechanism even approximates the objective of maximizing the winner's value subject to a minimum quality; we also show that the convexity assumption in Section 3 is necessary. Finally, in Section 5, we extend our mechanisms and characterization to a much more general setting.

Related work. To the best of our knowledge, our work is the first to address mechanism design in a market for daily deals. There has been unrelated work on other aspects of daily deals (*e.g.* impact on reputation) [4, 5, 14]. A related, but different line of work deals with mechanism design for pay-per-click (PPC) advertising. In that setting, as in ours, each ad has a value and a quality (representing click-through rate for PPC ads and the probability of purchasing the deal in our setting). The objective is often to maximize the combined utility of the advertisers and the auctioneer [18, 9], but variants where the utility of the user is also taken into account have also been studied [1]. The crucial difference is that in PPC advertising, the auctioneer holds the quality parameter, whereas in our setting, this parameter is only known to the merchant and truthful extraction of the parameter is an important part of the problem. Other work on auctions with a quality component [6, 10] assume that a quality level may be assigned by the mechanism to the bidder (who always complies), in contrast to our setting where quality is fixed and private information.

We make use of proper scoring rules, an overview of which appears in [11]; to our knowledge, proper scoring rules have been used in auctions only to incentivize agents to guess others' valuations [2]. Our general setting is related to an extension of proper scoring rules, decision rules and decision markets [13, 15]. There, a mechanism designer elicits agents' predictions of an event conditional on which choice she makes. She then selects an outcome, observes the event, and pays the agents according to the accuracy of their predictions. Unlike our setting, agents are assumed not to have preferences over the designer's choice, except in [3], which (unlike us) assumes that the mechanism has partial knowledge of these preferences and does not attempt to elicit preferences. Our general model may be interpreted as a fully general extension to the decision-rule setting in which we introduce the novel challenge of *truthfully eliciting* these preferences and incorporate them into the objective. However, we focus on deterministic mechanisms, while randomized mechanisms have been shown to have nice properties in a decision-rule setting [7].

Another related line of work examines examines when a proper scoring rule might incentivize an agent to take undesirable actions in order to improve his prediction's accuracy. When the mechanism designer has preferences over different states, scoring rules that incentivize beneficial actions are termed *principal-aligned* scoring rules [17]. A major difference is that the mechanism designer in the principal-aligned setting, unlike in ours, does not select between outcomes of any mechanism, but merely observes a state of the world and makes payments.

2 The Model

In this section, we formulate the problem in its simplest form: when an auctioneer has to select just one of the interested merchants to display her coupon to a single consumer.[1] In Section 5, our model and results will be generalized to a much broader setting.

There are m bidders, each with a single coupon. We also refer to the bidders as *merchants* and to coupons as *deals*. An auctioneer selects at most one of these coupons to display. For each bidder i, there is a probability $p_i \in [0, 1]$ that if i's coupon is displayed to a consumer, it will be purchased

[1] Our mechanisms for this model can be immediately extended to the case of many consumers by scaling.

by the consumer. We refer to p_i as the *quality* of coupon i. Furthermore, for each bidder i, there is a value $v_i \in \mathbb{R}$ that represents the expected value that i gets if her coupon is chosen to be displayed to the advertiser. Both v_i and p_i are private information of the bidder i, and are unknown to the auctioneer.[2] We refer to (v_i, p_i) as bidder i's *type*. We assume that the bidders are expected utility maximizers and their utility is quasilinear in payment.

Note that v_i is i's total expected valuation for being selected; in particular, it is *not* a value-per-purchase (as in *e.g.* search advertisement). Rather, v_i is the maximum amount i would be willing to pay to be selected (before observing the consumer's purchasing decision). Also, we allow v_i and p_i to be related in an arbitrary manner. If, for instance, i derives value a_i from displaying the coupon plus an additional c_i if the consumer purchases the coupon, then i would compute $v_i = a_i + p_i c_i$ and submit her true type (v_i, p_i). For our results, we do not need to assume any particular model of how v_i is computed or of how it relates to p_i.

An auction mechanism functions as follows. It asks each bidder i to reveal her private type (v_i, p_i). Let $(\hat{v}_i, \hat{p}_i)$ denote the type reported by bidder i. Based on these reports, the mechanism chooses one bidder i^* as the *winner* of the auction, i.e., the merchant whose deal is shown. Then, a consumer arrives; with probability p_{i^*}, she decides to purchase the deal. Let $\omega \in \{0, 1\}$ denote the consumer's decision (where 1 is a purchase). The mechanism observes the consumer's decision and then charges the bidders according to a payment rule, which may depend on ω.

We require the mechanism to be *truthful*, which means that it is, first, *incentive compatible*: for every merchant i and every set of types reported by the other merchants, i's expected utility is maximized if she reports her true type (v_i, p_i); and second, *interim individually rational*: each merchant receives a non-negative utility in expectation (over the randomization involved in the consumer's purchasing decision) if she reports her true type.

The goal of the auctioneer is to increase some combination of the welfare of all the parties involved. If we ignore the consumer, this can be modeled by the sum of the utilities of the merchants and the auctioneer, which, by quasilinearity of the utilities, is precisely v_{i^*}. To capture the welfare of the user, we suppose that a reasonable proxy is the quality p_{i^*} of the selected deal. We study two natural ways to combine the merchant/auctioneer welfare v_{i^*} with the consumer welfare p_{i^*}. One is to maximize v_{i^*} subject to the deal quality

[2] In Section 5.4, we will briefly discuss extensions in which both parties have quality information.

p_{i^*} meeting a minimum threshold α. Another is to model the consumer's welfare as a function $g(p_{i^*})$ of quality and seek to maximize total welfare $v_{i^*} + g(p_{i^*})$. In the latter case, when g is a convex function, we construct in the next section a truthful mechanism that maximizes this social welfare function (and we show in Section 4 that, when g is not convex, there is no such mechanism). For the former case, in Section 4, we prove that it is not possible to achieve the objective, even approximately.

3 A Truthful Mechanism via Proper Scoring Rules

In this section, we show that for every *convex* function g, there is an incentive-compatible mechanism that maximizes the social welfare function $v_{i^*}+g(p_{i^*})$. A convex consumer welfare g function may be natural in many settings. Most importantly, it includes the natural special case of a linear function; and it also intuitively models *risk aversion*, because (by definition of convexity) the average welfare of taking a guaranteed outcome, which is $pg(1)+(1-p)g(0)$, is larger than the welfare $g(p)$ of facing a lottery over those outcomes.[3][4]

We will make use of *binary scoring rules*, which are defined as follows.

Definition 1. *A binary scoring rule $S : [0,1] \times \{0,1\} \mapsto \mathbb{R}$ is a function that assigns a real number $S(\hat{p},\omega)$ to each probability report $\hat{p} \in [0,1]$ and state $\omega \in \{0,1\}$. The expected value of $S(\hat{p},\omega)$, when ω is drawn from a Bernoulli distribution with probability p, is denoted by $S(\hat{p};p)$. A scoring rule S is* (strictly) proper *if, for every p, $S(\hat{p};p)$ is (uniquely) maximized at $\hat{p}=p$.*

Traditionally, proper binary scoring rules are used to truthfully extract the probability of an observable binary event from an agent who knows this probability: It is enough to pay the agent $S(\hat{p},\omega)$ when the agent reports the probability $\hat{p}$ and the state turns out to be ω. In our setting, obtaining truthful reports is not so straightforward: A bidder's report affects whether

[3] To see this, suppose 100 consumers arrive, and the welfare of each is the convex function $g(p)=p^2$. If 50 consumers see a deal with $p=0$ and 50 see a deal with $p=1$, the total welfare is $50(0)+50(1)=50$. If all 100 see a deal with $p=0.5$, the total welfare is $100\left(0.5^2\right)=25$. Under this welfare function, the "sure bet" of 50 purchases is preferable to the lottery of 100 coin flips.

[4] Note that risk aversion is often associated with *concave* functions. These are unrelated as they do *not* map probability distributions to welfare; they are functions $u:\mathbb{R}\to\mathbb{R}$ that map wealth to welfare. Concavity represents risk aversion in that setting because the welfare of a guaranteed payoff x, which is $u(x)$, is larger than the welfare of facing a draw from a distribution with probability x, which is $xu(1)+(1-x)u(0)$.

or not they win the auction as well as any scoring rule payment. However, the following theorem shows that, when the consumer welfare function g is convex, then a careful use of proper binary scoring rules yields an incentive-compatible auction mechanism.

Theorem 1. *Let $g : \mathbb{R} \to \mathbb{R}$ be a convex function. Then there is a truthful auction that picks the bidder i^* that maximizes $v_{i^*} + g(p_{i^*})$ as the winner.*

The proof of this theorem relies on the following lemma about proper binary scoring rules, which is well known and given, for example, in [11]. For the sake of completeness, we include a proof here.

Lemma 1. *Let $g : [0,1] \to \mathbb{R}$ be a (strictly) convex function. Then there is a (strictly) proper binary scoring rule S_g such that for every p, $S_g(p;p) = g(p)$.*

Proof. Let $g'(p)$ be a subgradient of g at point p, i.e., a value such that for all $q \in [0,1]$, $g(q) \geq g(p) + g'(p)(q-p)$.[5] Now define:

$$S_g(p,1) = g(p) + (1-p)g'(p)$$
$$S_g(p,0) = g(p) - pg'(p)\ .$$

We have that $S_g(p;p) = pS_g(p,1) + (1-p)S_g(p,0) = g(p)$. We now prove that S_g is a (strictly) proper binary scoring rule. We have

$$\begin{aligned} S_g(\hat{p};p) &= p\left(g(\hat{p}) + (1-\hat{p})g'(\hat{p})\right) + (1-p)\left(g(\hat{p}) - \hat{p}g'(\hat{p})\right) \\ &= g(\hat{p}) + (p-\hat{p})g'(\hat{p}). \end{aligned}$$

By definition of g', the above value is never greater than $g(p)$, and is equal to $g(p)$ at $\hat{p} = p$. Therefore, the maximum of $S_g(\hat{p}, p)$ is achieved at $\hat{p} = p$. If g is strictly convex, the inequality is strict whenever $\hat{p} \neq p$. □

Proof of Theorem 1. Let h be the following "adjusted value" function: $h(\hat{v}, \hat{p}) = \hat{v} + g(\hat{p})$. For convenience, rename the bidders so that bidder 1 has the highest adjusted value, bidder 2 the next highest, and so on. The mechanism deterministically gives the slot to bidder $1 = i^*$. All bidders except bidder 1 pay zero. Bidder 1 pays $h(\hat{v}_2, \hat{p}_2) - S_g(\hat{p}_1, \omega)$, where S_g is a proper binary scoring rule satisfying $S_g(p;p) = g(p)$ and ω is 1 if the customer purchases the coupon and 0 otherwise. The existence of this binary scoring rule is guaranteed by Lemma 1.

[5] If g is differentiable, g' must be the derivative of g. Even if g is not differentiable, convexity of g implies that a subgradient g' always exists.

We now show that the auction is truthful. If i bids truthfully and does not win, i's utility is zero. If i bids truthfully and wins, i's expected utility is

$$\begin{aligned} & v_i - h(\hat{v}_2, \hat{p}_2) + S_g(p_i; p_i) \\ = & h(v_i, p_i) - h(\hat{v}_2, \hat{p}_2). \end{aligned}$$

This expected utility is always at least 0 because i is selected as winner only if $h(v_i, p_i) \geq h(v_2, p_2)$. This shows that the auction is interim individually rational.

Now suppose that i reports $(\hat{v}_i, \hat{p}_i)$. If i does not win the auction with this report, then i's utility is zero, but a truthful report always gives at least zero. So we need only consider the case where i wins the auction with this report. Then, i's expected utility is

$$\begin{aligned} & v_i - h(\hat{v}_2, \hat{p}_2) + S_g(\hat{p}_i; p_i) \\ \leq & v_i - h(\hat{v}_2, \hat{p}_2) + S_g(p_i; p_i) \\ = & h(v_i, p_i) - h(\hat{v}_2, \hat{p}_2). \end{aligned}$$

using the properness of S_g and the definition of $h(v_i, p_i)$. There are two cases. First, if $h(v_i, p_i) < h(\hat{v}_2, \hat{p}_2)$, then $U(\hat{v}_i, \hat{p}_i) < 0$. But, if i had reported truthfully, i would have gotten a utility of zero (having not have been selected as the winner). Second, if $h(v_i, p_i) \geq h(\hat{v}_2, \hat{p}_2)$, then $U(\hat{v}_i, \hat{p}_i) \leq h(v_i, p_i) - h(\hat{v}_2, \hat{p}_2)$. But, if i had reported truthfully, i would have gotten an expected utility of $h(v_i, p_i) - h(\hat{v}_2, \hat{p}_2)$. This shows incentive compatibility. □

4 Impossibility Results

An alternative way to combine consumer welfare with the advertiser/auctioneer welfare is to ask for an outcome that maximizes the advertiser/auctioneer welfare subject to the winner's quality parameter meeting a minimum threshold α. It is not hard to show that achieving such "discontinuous" objective functions is impossible.[6] A more reasonable goal is to obtain an incentive-compatible mechanism with the following property: for two given thresholds α and β with $\alpha < \beta$, the mechanism always selects a winner i^* with quality p_{i^*} at least α, and with a value v_{i^*} that is at least $v^* := \max_{i:p_i \geq \beta}\{v_i\}$ (or

[6]Intuitively, the reason is that it is impossible to distinguish between a coin whose probability of heads is α and one whose probability is $\alpha - \epsilon$, when ϵ can be arbitrarily small, by the result of a single flip.

an approximation of v^*).

One approach to solving this problem is to use the result of the previous section (Theorem 1) with an appropriate choice of the function g. Indeed, if we assume the values are from a bounded range $[0, V_{\max})$ and use the auction mechanism from Theorem 1 with a function g defined as follows,

$$g(p) = \begin{cases} 0 & \text{if } p < \alpha \\ \frac{p-\alpha}{\beta-\alpha}.V_{\max} & \text{if } p \geq \alpha \end{cases}$$

then if there is at least one bidder with quality parameter at least β, then the mechanism is guaranteed to pick a winner with quality at least α. This is easy to see: the adjusted bid of the bidder with quality at least β is at least $V_{\max}$, while the adjusted bid of any bidder with quality less than α is less than $V_{\max}$. In terms of the value, however, this mechanism cannot provide any multiplicative approximation guarantee, as it can select a bidder with quality 1 and value 0 over a bidder with quality β and any value less than $\frac{1-\alpha}{\beta-\alpha}V_{\max}$.

Unfortunately, as we show in Theorem 2:, this is unavoidable: unless $\beta = 1$ (that is, unless welfare is compared only against bidders of "perfect" quality), there is no deterministic, truthful mechanism that can guarantee a bounded multiplicative approximation guarantee in the above setting.

Theorem 2. *For a given $0 \leq \alpha < \beta \leq 1$ and $\lambda \geq 1$, suppose that a deterministic truthful mechanism satisfies that, if there is some bidder i with $p_i \geq \beta$:*

1. *The winner has $p_{i^*} > \alpha$;*

2. *The winner has value $v_{i^*} \geq v^*/\lambda$, where $v^* := \max_{i:p_i \geq \beta}\{v_i\}$.*

Then $\beta = 1$. This holds even if valuations are upper-bounded by a constant V_{max}.

Proof. Fix all reports $\vec{v}_{-i}$ and $\vec{p}_{-i}$. Let $t_\omega(\hat{v}_i, \hat{p}_i)$ be the net transfer to bidder i in state ω when i reports $(\hat{v}_i, \hat{p}_i)$ and wins the auction ($t_\omega(\hat{v}_i, \hat{p}_i)$ will be negative if the mechanism charges bidder i). Then we can denote i's expected utility for winning with report $(\hat{v}_i, \hat{p}_i)$ given true type (v_i, p_i) by

$$U(\hat{v}_i, \hat{p}_i; v_i, p_i) = v_i + p_i t_1(\hat{v}_i, \hat{p}_i) + (1 - p_i)t_0(\hat{v}_i, \hat{p}_i) \ .$$

Since i will always report so as to maximize this value given that i prefers to win, we can define

$$h(p_i) = \max_{(\hat{v}_i, \hat{p}_i) \in \mathcal{W}} \{p_i t_1(\hat{v}_i, \hat{p}_i) + (1 - p_i)t_0(\hat{v}_i, \hat{p}_i)\},$$

where $\mathcal{W}$ is the set of winning bids $(\hat{v}_i, \hat{p}_i)$, and write i's expected utility for winning simply as $U(v_i, p_i) = \max_{(\hat{v}_i, \hat{p}_i) \in \mathcal{W}} U(\hat{v}_i, \hat{p}_i; v_i, p_i) = v_i + h(p_i)$. We note that h is a convex function of p since, for any pricing scheme $t_\omega(\hat{v}_i, \hat{p}_i)$, $h(p)$ is the point-wise maximum over a family of linear functions.

Fix some choices of $0 \leq \alpha < \beta \leq 1$. To guarantee that i does not win if $p_i \leq \alpha$, we must have that, whenever $p_i \leq \alpha$, every winning bid gives i negative expected utility. Therefore, i will not bid so as to win in this case. Thus,

$$\begin{aligned} & U(v_i, p_i) < 0 && (\forall v_i, p_i \leq \alpha) \\ \implies & h(\alpha) < -V_{max} \ . \end{aligned}$$

Now, suppose there is a v_1 with the property that i is never selected as winner when $v_i < v_1$. Then we must have

$$\begin{aligned} & U(v_i, p_i) < 0 && (\forall v_i < v_1, p_i) \\ \implies & h(p_i) < -v_1 && (\forall p_i) \ . \end{aligned}$$

Conversely, suppose that there is a v_2 with the property that i is always selected as the winner when $p_i \geq \beta$ and $v_i > v_2$. Then we must have

$$\begin{aligned} & U(v_i, p_i) > 0 && (\forall v_i > v_2, p_i \geq \beta) \\ \implies & h(\beta) > -v_2 \ . \end{aligned}$$

Since h is convex,

$$\begin{aligned} \left(\frac{\beta - \alpha}{1 - \alpha}\right) h(1) + \left(\frac{1 - \beta}{1 - \alpha}\right) h(\alpha) &\geq h\left(\frac{\beta - \alpha}{1 - \alpha} + \frac{1 - \beta}{1 - \alpha}(\alpha)\right) \\ &= h(\beta) \ . \end{aligned}$$

The above inequalities thus imply that

$$v_2 \geq V_{max}\left(\frac{1 - \beta}{1 - \alpha}\right) + v_1\left(\frac{\beta - \alpha}{1 - \alpha}\right) \ . \tag{1}$$

Now suppose that our mechanism guarantees a welfare approximation factor of λ. Let v^* be the highest value of any bidder other than i having $p \geq \beta$ (supposing such a bidder exists). Then i loses if $v_i < v^*/\lambda = v_1$ and wins whenever $p_i \geq \beta$ and $v_i > \lambda v^* = v_2$. But v_1 and v_2 satisfy the properties given above, so they satisfy Inequality 1. Now take v^* arbitrarily small, so that $v_1, v_2 \ll V_{max}$, and Inequality 1 can only hold if $\beta = 1$. □

The techniques used in the above proof can be used to show that the convexity assumption in Theorem 1 is indeed necessary:

Theorem 3. *Assume $g : \mathbb{R} \to \mathbb{R}$ is a function for which there exists a deterministic truthful auction that always picks the bidder i^* that maximizes $v_{i^*} + g(p_{i^*})$ as the winner. Then g is a convex function.*

Proof. As in the proof of Theorem 2, fix all reports $\vec{v}_{-i}$ and $\vec{p}_{-i}$, and define $t_\omega(\hat{v}_i, \hat{p}_i)$ and $U(\hat{v}_i, \hat{p}_i; v_i, p_i)$ as before. By the incentive compatibility and individual rationality of the mechanism, bidder i must win the auction if

$$\max_{(\hat{v}_i, \hat{p}_i)} U(\hat{v}_i, \hat{p}_i; v_i, p_i) > 0$$

and lose if

$$\max_{(\hat{v}_i, \hat{p}_i)} U(\hat{v}_i, \hat{p}_i; v_i, p_i) < 0.$$

Equivalently, bidder i must win if

$$v_i > -\max_{(\hat{v}_i, \hat{p}_i)} \{p_i t_1(\hat{v}_i, \hat{p}_i) + (1 - p_i) t_0(\hat{v}_i, \hat{p}_i)\}$$

and lose if the opposite inequality holds. On the other hand, since the mechanism always picks the bidder that maximizes $v_i + g(p_i)$, bidder i must win if

$$v_i > \max_{j \neq i} \{v_j + g(p_j)\} - g(p_i)$$

and lose if the opposite inequality holds. Thus, we must have:

$$\begin{aligned} &\max_{j \neq i} \{v_j + g(p_j)\} - g(p_i) = \\ &\qquad -\max_{(\hat{v}_i, \hat{p}_i)} \{p_i t_1(\hat{v}_i, \hat{p}_i) + (1 - p_i) t_0(\hat{v}_i, \hat{p}_i)\}, \end{aligned}$$

or

$$g(p_i) = \max_{(\hat{v}_i, \hat{p}_i)} \{p_i t_1(\hat{v}_i, \hat{p}_i) + (1 - p_i) t_0(\hat{v}_i, \hat{p}_i)\} + \max_{j \neq i} \{v_j + g(p_j)\}.$$

The right-hand side of the above equation is the maximum of a number of terms, each of which is a linear function of p_i. Therefore, $g(p_i)$ is a convex function of p_i. □

5 A General Framework

Daily deals websites generally offer many deals simultaneously, and to many consumers. A more realistic model of this scenario must take into account complex *valuation functions* as well as general *quality reports*. Merchants' valuations may depend on which slot (top versus bottom, large versus small) or even *subset* of slots they win; they may also change depending on which competitors are placed in the other slots. Meanwhile, merchants might like to report quality in different units than purchase probability, such as (for example) total number of coupon sales in a day, coupon sales relative to those of competitors, or so on.

In this section, we develop a general model that can cover these cases and considerably more. As in a standard multidimensional auction, bidders have a valuation for each outcome of the mechanism (for instance, each assignment of slots to bidders). For quality reports, our key insight is that they may be modeled by a *belief* or *prediction* over possible states of the world, where each state has some verifiable quality. This naturally models many scenarios where the designer would like to make a social choice (such as allocating goods) based not only on the valuations of the agents involved, but also on the likely externality on some non-bidding party; however, this externality can be best estimated by the bidders. We model this externality by a function, which we call the *consumer welfare function*, that maps probability distributions to a welfare value. A natural consumer welfare function is the expected value of a distribution.

When this consumer welfare satisfies a convexity condition, we construct truthful mechanisms for welfare maximization in this general setting; we also prove matching negative results. This allows us to characterize implementable welfare functions in terms of *component-wise convexity*, which includes the special case of expected value and can also capture intuitively risk-averse preferences.

We start with a definition of the model in Section 5.1, and then give a truthful mechanism as well as a matching necessary condition for implementability in this model in Section 5.2. In Section 5.4 we give a number of applications and extensions of our general framework.

5.1 Model

We now define the general model, using the multi-slot daily deals problem as a running example to illustrate the definition.

There are m *bidders* (also called *merchants*) indexed 1 through m, and a finite set $\mathcal{O}$ of possible *outcomes* of the mechanism. Each bidder has as private information a valuation function $v_i : \mathcal{O} \to \mathbb{R}$ that assigns a value $v_i(o)$ to each outcome o. For instance, each outcome o could correspond to an assignment of merchants to the available slots, and $v_i(o)$ is i's expected value for this assignment, taking into account the slot(s) assigned to i as well as the coupons in the other slots.

For each $o \in \mathcal{O}$ and each bidder i, there is a finite set of observable disjoint *states* of interest $\Omega_{i,o}$ representing different events that could occur when the mechanism's choice is o. For example, if merchant i is awarded a slot under outcome o, then $\Omega_{i,o}$ could be the possible total numbers of sales of i's coupon when the assignment is o, *e.g.* $\Omega_{i,o} = \{$fewer than 1000, 1000 to 5000, more than 5000$\}$.

Given an outcome o chosen by the mechanism, nature will select at random one of the states ω in $\Omega_{i,o}$ for each bidder i.[7] In the running example, some number of consumers choose to purchase i's coupon, so perhaps $\omega =$ "1000 to 5000".

We let $\Delta_{\Omega_{i,o}}$ denote the probability simplex over the set $\Omega_{i,o}$, i.e., $\Delta_{\Omega_{i,o}} = \{p \in [0,1]^{\Omega_{i,o}} : \sum_{\omega \in \Omega_{i,o}} p_\omega = 1\}$. Each bidder i holds as private information a set of beliefs (or predictions) $p_i : \mathcal{O} \to \Delta_{\Omega_{i,o}}$. For each outcome o, $p_i(o) \in \Delta_{\Omega_{i,o}}$ is a probability distribution over states $\omega \in \Omega_{i,o}$. Thus, under outcome o where i is assigned a slot, $p_i(o)$ would give the probability that i sells fewer than 1000 coupons, that i sells between 1000 and 5000 coupons, and that i sells more than 5000 coupons. We denote the vector of predictions $(p_1(o), \ldots, p_m(o))$ at outcome o by $\vec{p}(o) \in \times_{i=1}^m \Delta_{\Omega_{i,o}}$.

The goal of the mechanism designer is to pick an outcome that maximizes a notion of welfare. The combined welfare of the bidders and the auctioneer can be represented by $\sum_{i=1}^m v_i(o)$. If this was the goal, then the problem could have been solved by ignoring the $p_i(o)$'s and using the well-known Vickrey-Clarke-Groves mechanism [19, 8, 12]. In our setting, however, there is another component in the welfare function, which for continuity with the daily deals setting we call the *consumer welfare*. This component, which depends on the probabilities $p_i(o)$, represents the welfare of a non-bidding party that the auctioneer wants to keep happy (which could even be the auctioneer herself!). The consumer welfare when the mechanism chooses outcome o is given by an arbitrary function $g_o : \times_{i=1}^m \Delta_{\Omega_{i,o}} \to \mathbb{R}$ which depends on the bidders' predictions $\vec{p}(o)$. The goal of the mechanism designer is then to pick an outcome o that maximizes

$$\left(\sum_{i=1}^m v_i(o)\right) + g_o(\vec{p}(o)).$$

For example, in the multi-slot problem, consumer welfare at the outcome

[7] These choices do not have to be independent across bidders; indeed, all bidders could be predicting the same event, in which case $\Omega_{i,o} = \Omega_{i',o}$ for all i, i' and nature selects the same state for each i.

o could be defined as the sum of the expected number of clicks of the deals that are allocated a slot in o.

A mechanism in this model elicits bids $(\hat{v}_i, \hat{p}_i)$ from each bidder i and picks an outcome o based on these bids. Then, for each i, the mechanism observes the state ω_i picked by nature from $\Omega_{i,o}$ and charges i an amount that can depend on the bids as well as the realized state ω_i. This mechanism is *truthful* (incentive compatible and individually rational) if, for each bidder i, and for any set of reports of other bidders $(\hat{v}_{-i}, \hat{p}_{-i})$, bidder i can maximize her utility by bidding her true type (v_i, p_i), and this utility is non-negative.

5.2 A Class of Truthful Mechanisms

In this section, we give a truthful mechanism for the general setting, assuming that the consumer welfare function g_o satisfies the following convexity property.

Definition 2. *A function $f : \Delta_\Omega \mapsto \mathbb{R}$ is convex if and only if for each $x, y \in \Delta_\Omega$ and each $\alpha \in [0,1]$,*

$$f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y).$$

We call a function $g_o : \times_{i=1}^m \Delta_{\Omega_{i,o}} \mapsto \mathbb{R}$ component-wise convex *if for each i and for each vector $\vec{p}_{-i}(o) \in \times_{j:j\neq i} \Delta_{\Omega_{j,o}}$ of predictions of bidders other than i, $g_o(p_i(o), \vec{p}_{-i}(o))$ is a convex function of $p_i(o)$.*

Component-wise convexity includes the important special case of expected value, and can also capture an intuitive notion of risk aversion with respect to each bidder's prediction, as it requires that the value of taking a draw from some distribution gives lower utility than the expected value of that draw (see footnotes 3 and 4). It includes all convex functions, but there are also functions such as $g(p_1, p_2) = \left(p_1 - \frac{1}{2}\right) \cdot \left(p_2 - \frac{1}{2}\right)$ that are component-wise convex but not convex.

We can now state our result.

Theorem 4. *If for any outcome o, the consumer welfare function g_o is component-wise convex, then there is a truthful mechanism that selects an outcome o that maximizes $(\sum_{i=1}^m v_i(o)) + g_o(\vec{p}(o))$.*

As in the simple model, our mechanism uses proper scoring rules. The definition of scoring rules can be adapted to the general setting as follows.

Definition 3. *A* scoring rule *$S : \Delta_\Omega \times \Omega \to \mathbb{R}$ is a function that assigns a real number $S(p, \omega)$ to each probability report $p \in \Delta_\Omega$ and state $\omega \in \Omega$.*

The expected value of $S(\hat{p},\omega)$ *when* ω *is drawn according to the distribution* $p \in \Delta_\Omega$ *is denoted by* $S(\hat{p};p)$*. A scoring rule* S *is* proper *if, for every* p*,* $S(\hat{p};p)$ *is maximized at* $\hat{p}=p$*.*

We also need a generalization of Lemma 1. This fact about scoring rules is proven in more generality in [11] and is originally due to Savage [16].

Lemma 2 ([11, 16])**.** *For every convex function* $g : \Delta_\Omega \to \mathbb{R}$ *there is a proper scoring rule* S_g *such that for every* p*,* $S_g(p;p) = g(p)$*.*

Proof of Theorem 4. Using Lemma 2 and the assumption that g_o is componentwise convex, for any outcome o, bidder i, and set of reports of other bidders $\vec{p}_{-i}(o)$, we can construct a proper scoring rule $S_{o,i,\vec{p}_{-i}(o)}$ with $S_{o,i,\vec{p}_{-i}(o)}(p_i(o);p_i(o)) = g_o(p_i(o),\vec{p}_{-i}(o))$. The function $S_{o,i,\vec{p}_{-i}(o)}$ scores prediction $\hat{p}_i(o)$ on states $\omega \in \Omega_{i,o}$.

Next, we use a Vickrey-Clark-Groves-like mechanism and show that truthfulness is a dominant strategy. Let $W^o = \sum_{i=1}^m v_i(o) + g_o(\vec{p}(o))$, where (v_i,p_i) is the bid of bidder i. Our mechanism selects outcome o^* that maximizes W^{o^*}. Let W_{-i} be the value of the selection made by our mechanism on the set of bids excluding i. Each bidder i, when state $\omega \in \Omega_{i,o^*}$ occurs, pays

$$W_{-i} - \sum_{i' \neq i} v_{i'}(o^*) - S_{o^*,i,\vec{p}_{-i}(o^*)}(p_i(o^*),\omega).$$

Therefore, under outcome o^*, bidder i's expected utility for reporting truthfully is

$$\begin{aligned} U(v_i,p_i) &= v_i(o^*) - W_{-i} + \sum_{i' \neq i} v_{i'}(o^*) + S_{o^*,i,\vec{p}_{-i}(o^*)}(p_i(o^*);p_i(o^*)) \\ &= \sum_{i'=1}^{m} v_{i'}(o^*) - W_{-i} + g_{o^*}(\vec{p}(o^*)) \\ &= W^{o^*} - W_{-i} \end{aligned} \tag{2}$$

The above value is clearly non-negative. Now, consider a scenario where i changes her bid to $(\hat{v}_i,\hat{p}_i)$, when her type is still given by (v_i,p_i). Let o' denote the outcome selected in that scenario. The utility of bidder i in this scenario can be written as:

$$\begin{aligned}
U(\hat{v}_i, \hat{p}_i) &= v_i(o') - W_{-i} + \sum_{i' \neq i} v_{i'}(o') + S_{o',i,\vec{p}_{-i}(o')}(\hat{p}_i(o'); p_i(o')) \\
&\leq \sum_{i'=1}^{m} v_{i'}(o') - W_{-i} + g_{o'}(\vec{p}(o')) \\
&= W^{o'} - W_{-i}, \qquad (3)
\end{aligned}$$

where the first inequality follows from the fact that $S_{o',i,\vec{p}_{-i}(o')}$ is a proper scoring rule with $S_{o',i,\vec{p}_{-i}(o')}(p_i(o'); p_i(o')) = g_{o'}(\vec{p}(o'))$.

Finally, note that by the definition of o^*, we have $W^{o^*} \geq W^{o'}$. This inequality, together with (2) and (3) implies that $U(\hat{v}_i, \hat{p}_i) \leq U(v_i, p_i)$. Therefore, i cannot gain by misreporting her type. □

5.3 Characterization of Implementable Consumer Welfare Functions

In this section, we show that the *component-wise convexity* assumption that we imposed on the consumer welfare function in the last section to derive a truthful mechanism is indeed necessary. In other words, in our general setting, the consumer welfare functions that are implementable using dominant-strategy truthful mechanisms are precisely those that are component-wise convex.

Theorem 5. *Suppose g is a consumer welfare function and there exists a deterministic truthful mechanism that always selects the outcome o that maximizes $\sum_{i=1}^{m} v_i(o) + g_o(\vec{p}(o))$. Then g_o is component-wise convex for every o.*

Proof. Fix a bidder i and bids (v_{-i}, p_{-i}) of all the other bidders. We prove that for every outcome o, the function $g_o(\vec{p}(o))$ as a function of $p_i(o)$ is convex. This shows that g_o is component-wise convex for every o.

For any bid $(\hat{v}_i, \hat{p}_i)$ for bidder i, the mechanism selects an outcome o, and charges i an amount depending on the realized state $\omega \in \Omega_{i,o}$. This payment can be represented by a vector in $\mathbb{R}^{\Omega_{i,o}}$ (a negative value in this vector indicates a value that the bidder pays the auctioneer, and a positive value indicates a reverse transfer). Let $A_{o,i,v_{-i},p_{-i}} \subseteq \mathbb{R}^{\Omega_{i,o}}$ denote the collection of payment vectors corresponding to all bids $(\hat{v}_i, \hat{p}_i)$ for bidder i that (along with the bids (v_{-i}, p_{-i}) for others) result in the mechanism picking outcome o. Since we have fixed i and v_{-i}, p_{-i}, we simply denote this collection by A_o.

The utility of i when she submits a bid that results in outcome o and payment $t \in A_o$ can be written as $v_i(o)+t.p_i(o)$ (the latter term is the inner product of $t \in \mathbb{R}^{\Omega_{i,o}}$ and $p_i(o) \in \Delta_{\Omega_{i,o}}$). By truthfulness of the mechanism, i's utility from truthful bidding must be

$$\max_{o\in\mathcal{O}} \max_{t\in A_o}\{v_i(o) + t.p_i(o)\},$$

and the outcome selected by the mechanism must be the o that maximizes the above expression. Denoting

$$f_o(p_i(o)) = \max_{t\in A_o}\{t.p_i(o)\}, \tag{4}$$

this means that the mechanism selects the outcome o if

$$v_i(o) + f_o(p_i(o)) > \max_{o'\neq o}\{v_i(o') + f_{o'}(p_i(o'))\}$$

and does not select this outcome if the reverse inequality holds. This means that holding everything other than $v_i(o)$ constant, the threshold for $v_i(o)$ after which the mechanism selects the outcome o is precisely

$$\max_{o'\neq o}\{v_i(o') + f_{o'}(p_i(o'))\} - f_o(p_i(o)). \tag{5}$$

On the other hand, the mechanism always picks an outcome o that maximizes $\sum_{j=1}^m v_j(o) + g_o(\vec{p}(o))$. Therefore, holding everything except $v_i(o)$ constant, the threshold for $v_i(o)$ after which the outcome o is selected is precisely

$$\max_{o'\neq o}\left\{\sum_{j=1}^m v_j(o') + g_{o'}(\vec{p}(o'))\right\} - \sum_{j\neq i} v_j(o) - g_o(\vec{p}(o)). \tag{6}$$

Therefore, the thresholds (5) and (6) must be equal. Writing this equality, and moving $g_o(\vec{p}(o))$ to the left-hand side of the equality and everything else to the right-hand side, we obtain:

$$\begin{aligned} g_o(\vec{p}(o)) = &-\max_{o'\neq o}\{v_i(o') + f_{o'}(p_i(o'))\} + f_o(p_i(o)) \\ &+ \max_{o'\neq o}\left\{\sum_{j=1}^m v_j(o') + g_{o'}(\vec{p}(o'))\right\} - \sum_{j\neq i} v_j(o). \end{aligned}$$

Now, observe that the only term on the right-hand side of the above equation that depends on $p_i(o)$ is $f_o(p_i(o))$. Furthermore, $f_o(p_i(o))$ (as defined in Equation (4)) is the maximum over a family of linear functions of $p_i(o)$, and therefore is a convex function of $p_i(o)$. This means that fixing any set of values for $\vec{p}_{-i}(o)$, $g_o(\vec{p}(o))$ is a convex function of $p_i(o)$. Therefore g_o is component-wise convex. □

5.4 Applications

In this section, we present a few sample applications and extensions of our general framework. This demonstrates that the results of Section 5.2 can be used to characterize achievable objective functions and design truthful mechanisms in a very diverse range of settings.

Daily Deals with Both Merchant and Platform Information. In some cases, it might be reasonable in a daily deals setting to suppose that the platform, as well as the merchant, has some relevant private information about deal quality. For example, perhaps the merchant has specific information about his particular deal, while the auctioneer has specific information about typical consumers under particular circumstances (days of the week, localities, and so on). Many such extensions are quite straightforward; intuitively, this is because we solve the difficult problem: incentivizing merchants to truthfully reveal quality information.

To illustrate, consider a simple model where merchant i gets utility a_i from displaying a deal to a consumer and an additional c_i if the user purchases it. For every assignment of slots o containing the merchant's deal, its quality (probability of purchase) is a function $f_{o,i}$ of two pieces of private information: x_i, held by the merchant, and y_i, held by the platform. Each merchant is asked to submit (a_i, c_i, x_i). The platform computes, for each slot assignment o, $p_i(o) = f_{o,i}(x_i, y_i)$, then sets $v_i(o) = a_i + p_i(o)c_i$ for all o that include i's coupon ($v_i(o) = 0$ otherwise). Then, the platform runs the auction defined in Theorem 4, setting i's bid equal to (v_i, p_i). By Theorem 4, bidder i maximizes expected utility when v_i is her true valuation for winning and p_i is her true deal quality; therefore, she can maximize expected utility by truthfully submitting (a_i, c_i, x_i), as this allows the mechanism to correctly compute v_i and p_i.

Reliable Network Design. Consider a graph G, where each edge is owned by a different agent. The auctioneer wants to buy a path from a source node s to a destination node t. Each edge has a cost for being used in the path, and also a probability of failure. Both of these parameters are private values of the edge. The goal of the mechanism designer is to buy a path from s to t that minimizes the total cost of the edges plus the cost of failure, which is a fixed constant times the probability that at least one of the edges on the path fails.

It is easy to see that the above problem fits in our general framework: each bidder's value is the negative of the cost of the edge; each "outcome" is a path from s to t; for each edge i on a path, the corresponding "states" are fail and succeed; the consumer welfare function g_o for an outcome o is the negative of the failure cost of that path. For each edge, fixing all other reports, g_o is a linear function of failure probability. Therefore, g_o is component-wise convex, and Theorem 4 gives a truthful mechanism for this problem.

We can also model a scenario where each edge has a probabilistic delay instead of a failure probability. When edge i is included in the path, the possible states $\Omega_{i,o}$ correspond to the possible delays experienced on that edge. A natural objective function is to minimize the total cost of the path from s to t plus its expected delay, which is a linear function of probability distributions. We can also implement costs that are concave functions of the delay on each edge (as welfare, the negative of cost, is then convex). These model risk aversion, as, intuitively, the cost of a delay drawn from a distribution is higher than the cost of the expected delay of that distribution. (Note that our results imply that a *concave* objective function is *not* implementable!)

The exact same argument shows that other network design problems fit in our framework. For example, the goal can be to pick a k-flow from s to t, or a spanning tree in the graph. The "failure" function can also be more complicated, although we need to make sure the convexity condition is satisfied.

Principal-Agent Models with Probabilistic Signals. Another application of our mechanism is in a *principal-agent* setting, where a principal would like to incentivize agents to exert an optimal level of effort, but can only observe a probabilistic signal of this effort. Suppose the principal wishes to hire a set of agents to complete a project; the principal only observes whether each agent succeeds or fails at his task, but the probability of each's success is influenced by the amount of effort he puts in. More precisely, let $c_i(e)$ denote the cost of exerting effort e for agent i and $p_i(e)$ denote the probability of the agent's success if this agent is hired and exerts effort e. The welfare generated by the project is modeled by a component-wise convex function of the agents' probabilities of success (for instance, a constant times the probability that all agents succeed).

At the first glance, it might seem that this problem does not fit within our framework, since each agent can affect its success probability by exerting more or less effort. However, suppose we define an outcome of the mechanism as selecting both a set of agents and an assignment of effort levels to these agents. Each agent submits as his type the cost $c_i(e)$ and probability of success $p_i(e)$ for each possible effort level e. Theorem 4 then gives a welfare-maximizing mechanism that truthfully elicits the $c_i(e)$ and $p_i(e)$ values from each agent and selects the agents to hire, the effort levels they should exert, and the payment each receives conditional on whether his component of the project succeeds. Agents maximize expected utility by declaring their true types and exerting the amount of effort they are asked to.[8]

6 Conclusion

Markets for daily deals present a challenging new mechanism-design setting, in which a mechanism designer (the platform) wishes to pick an outcome (merchant and coupon to display) that not only gives good bidder/auctioneer welfare, but also good welfare for a third party (the consumer); however, this likely consumer welfare is private information of the bidders.

Despite the asymmetry of information, we show that, when the consumer welfare function is a convex function of bidders' quality, we can design truthful mechanisms for social welfare maximization in this setting. We give a matching negative result showing that no truthful, deterministic mechanism exists when consumer welfare is not convex. Another natural objective, approximating welfare subject to meeting a quality threshold, also cannot be achieved in this setting.

Extending the daily deals setting to a more general domain yields a rich setting with many potential applications. We model this setting as an extension to traditional mechanism design: Now, agents have both preferences over outcomes and probabilistic beliefs conditional on those outcomes. The goal is to maximize social welfare including the welfare of a non-bidding party, modeled by a consumer welfare function taking probability distributions over states of the world to welfare.

A truthful mechanism must incentivize bidders to reveal their true preferences *and* beliefs, even when these revealed beliefs influence the designer to pick a less favorable outcome for the bidders. We demonstrate that this is

[8]The proof is the same as that of truthfulness: If the agent deviates and exerts some other effort level, his expected utility will be bounded by if he had reported the truth and the mechanism had assigned him that effort level; but by design, this is less than his utility under the choice actually made by the mechanism.

possible if and only if the consumer welfare function is component-wise convex, and when it is, we explicitly design mechanisms to achieve the welfare objective. Component-wise convexity includes expected-welfare maximization and intuitively can capture risk averse preferences. Finally, we demonstrate the generality of our results with a number of example extensions and applications.

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