

Online Auctions for Bidders with Interdependent Values

Florin Constantin
HSEAS, Harvard University
33 Oxford St, Cambridge, MA
florin@eecs.harvard.edu

Takayuki Ito
Nagoya Institute of Technology
Nagoya 466-8555, Japan
itota@nitech.ac.jp

David C. Parkes
HSEAS, Harvard University
33 Oxford St, Cambridge, MA
parkes@eecs.harvard.edu

ABSTRACT

Interdependent values (IDV) is a valuation model allowing bidders in an auction to express their value for the item(s) to sell as a function of the other bidders' information. We investigate the incentive compatibility (IC) of single-item auctions for IDV bidders in *dynamic* environments. We provide a necessary and sufficient characterization for IC in this setting. We show that if bidders can misreport departure times and private signals, no reasonable auction can be IC. We present a reasonable IC auction for the case where bidders cannot misreport departures.

Categories and Subject Descriptors

J.4 [Social and Behavioral Sciences]: Economics

General Terms

Economics, Theory

Keywords

Interdependent, Dynamic Auctions, Incentive Compatibility

1. INTRODUCTION

The interest in dynamic auctions has surged over the last couple of years, motivated by their widespread use in electronic markets. A visible example is the eBay marketplace in which there are interesting dynamics both within and across auctions [7]. In most of these auctions, however, the interaction between bidder *valuations* is limited and only indirect, through the competition between bids. In many auctions such interaction is of interest, for instance when bidders have limited information about the value of the item(s) to be sold. A model from the auction theory literature that allows the bidders' values to be more sensitive to the information in the market is that of *interdependent values* (IDV): each bidder's value is defined as an aggregation (expressed via a *valuation function*) of the others' information, but he only knows his own private information (called his *signal*) [5].

Here is an example that illustrates our model. The item to sell is a rising Internet portal (such as YouTube); the

bidders are several Internet powerhouses (such as Yahoo! or Google). For example, a bidder may express its value as \$1.32 billion (the signal) + one fifth the maximum (or the average) signal among all the other bidders. Each bidder may become interested in buying the portal at a different time (construed as its arrival), and each bidder may have a different time (construed as its departure) at which it considers that it is more efficient for it to launch a portal of its own, and when its value for the portal drops to zero.

We introduce a model of interdependent valuations in *dynamic settings* and establish¹ a general negative result in this setting: there exists no individually rational, IC, reasonable dynamic auction for IDV bidders that are allowed elementary misreports. This stands in contrast with the private values environment [4]. Elementary misreports model bidders as being able to report any signal, any departure but an arrival no earlier than their true one. We present an IC reasonable auction for the case where bidders cannot misreport their departure.

Related work

Dasgupta and Maskin [2] provide an IC (static) auction that is efficient when bidders' signals are one dimensional. Ito and Parkes [6] instantiate this model to linear contingent bids and also extend it to single-minded combinatorial auctions. Aoyagi [1] investigates optimal pricing schemes in our framework. In his model however, bidders are assumed to be reporting honestly and a bidder's valuation may change based on past decisions of other bidders.

Hajiaghayi et al. [3, 4] provide competitive auctions in a dynamic environment, but they model bidders' values as private. The IC characterization that we provide in this paper generalizes similar characterizations that are provided in earlier work for private value environments [4, 8].

2. PRELIMINARIES

We assume that there is only one item for sale.² Each bidder i has a privately known *activity interval*: its bounds are the agent's arrival a_i and departure d_i . Each bidder i has private information s_i influencing its value for the item. We call s_i its *signal* and assume that it can be expressed as a one-dimensional, non-negative quantity. A bidder's *type* captures its private information: (arrival, departure, signal). Finally, each bidder has a *known* valuation function v_i that

¹All proofs are omitted in the interest of space and appear in the full version of the paper [9].

²Our results generalize immediately to settings with known supply of multiple units of an item and unit-demand bidders.

aggregates all signals available in the market and determines i 's actual value for the item.

For some time t denote by $A^{\leq t}$ (resp. $\theta^{\leq t}$) the signals (resp. types) of bidders that arrive at or before t . Similarly, if $a_i \leq t$ then $A_{-i}^{\leq t}$ (resp. $\theta_{-i}^{\leq t}$) denotes the signals (resp. types) of all bidders except i that arrive at or before t .

Bidder i 's value for the item for an allocation at time t is modeled as $v_i(s_i, A^{\leq d_i})$ at $t = d_i$, zero for $t > d_i$, but undefined for $t < d_i$. Before d_i the signal information from some bidders may not be yet revealed and thus i 's value is undefined. This is a significant change from the standard model for online, private-value auctions in which a bidder's valuation is known throughout his time in the auction. We model a bidder's value as zero after departure to indicate that a bidder is uninterested in an allocation after his departure.

For example, $v_i(s_i, A^{\leq d_i}) = 0.8s_i + 0.2 \max\{s_j : a_j \leq d_i\}$ if bidder i , whose signal is s_i , estimates the item's value to be the weighted average of s_i and the maximum signal of another bidder. In a private values setting, $v_i(s_i, A^{\leq d_i}) = s_i$.

We consider a model of *elementary misreports* in which the misreports available are arbitrary signal misreports coupled with *late* arrival misreports and arbitrary departure misreports. If one construes the arrival time as the period at which a bidder first learns of the existence of the auction or the time at which a bidder's signal is realized, whichever is latest, then it becomes nonsensical to consider early reports. As an auction designer we wish to provide incentives for a bidder to share that signal with other bidders.

Let bidder i 's true type be $(a_i, d_i; s_i)$, i 's reported type be $(a'_i, d'_i; s'_i)$ and let θ_{-i} denote the reported types from other bidders $\neq i$. A (deterministic) auction defines an allocation rule $q_i(a'_i, d'_i; s'_i, \theta_{-i}) \in \{0, 1\}$ to indicate whether or not bidder i is allocated the item, and a payment rule $p_i(a'_i, d'_i; s'_i, \theta_{-i}) \geq 0$ to define the payment made by bidder i . In a dynamic environment these must be online computable, i.e. $q_i(a'_i, d'_i; s'_i, \theta_{-i}) = q_i(a'_i, d'_i; s'_i, \theta_{-i}^{\leq d'_i})$ for all i , all θ , and similarly for payments. Moreover, payments must be collected by departure.

Bidders are modeled with quasilinear utilities: the utility of bidder i with type $(a_i, d_i; s_i)$ when reporting $(a'_i, d'_i; s'_i)$ is

$$q_i(a'_i, d'_i; s'_i, \theta_{-i})v_i(s_i, A_{-i}^{\leq d'_i}) - p_i(a'_i, d'_i; s'_i, \theta_{-i}).$$

Whatever his report, i 's true value for the item is $v_i(s_i, A_{-i}^{\leq d_i})$.

Given this model of self-interest we restrict attention to auctions having an equilibrium in which every bidder chooses to report his true type immediately upon arrival:

DEFINITION 1. *We say that an auction is (ex post) incentive compatible (IC, also known as truthful) if, for any truthfully reported types of other bidders, the ex post utility of any bidder is maximized if he reports his true type as well (i.e. truthful reporting is an ex post Nash equilibrium).*

DEFINITION 2. *We say that an auction is (ex post) individually rational (IR) if, for any truthfully reported types, losing bidders pay zero and the winner's payment is at most his true value for the item.*

Two assumptions are necessary for IC in static IDV auctions (see [2]) and therefore we adopt them as well: (1) *v-monotonicity*: a higher private signal cannot result in a lower value for the item; and (2) the *single crossing condition* (SCC): an infinitesimal change in bidder i 's private signal influences i 's value *more* than it influences the value of j if i 's value is equal to j 's and at least as high as the values of the other bidders.

3. IC CONSIDERATIONS

Let \mathbb{A} be a dynamic auction for IDV bidders. Fix bidder i and let (a'_i, d'_i, s'_i) denote his reported type and fix the reports of other bidders $\theta_{-i} = \theta_{-i}^{\leq d'_i}$.

The following conditions (that we will refer to as CAD) are *necessary and sufficient* for \mathbb{A} to be ex post IC and IR:

CRITICAL SIGNAL: Let

$$s_i^c[a'_i, d'_i, \theta_{-i}] = \inf\{s_i : i \text{ wins in } \mathbb{A} \text{ reporting } (a'_i, d'_i, s_i)\} \\ \text{and } \infty \text{ if no such } s_i \text{ exists (e.g. if the item has already been sold). Then when } s'_i > s_i^c[a'_i, d'_i, \theta_{-i}], \text{ bidder } i \\ \text{must win in } \mathbb{A} \text{ at price } v_i\left(s_i^c[a'_i, d'_i, \theta_{-i}], A_{-i}^{\leq d'_i}\right).$$

ARRIVAL MONOTONICITY: $s_i^c[a_i^+, d'_i, \theta_{-i}] \geq s_i^c[a'_i, d'_i, \theta_{-i}]$, $\forall a_i^+ \in (a'_i, d'_i]$

DEPARTURE MONOTONICITY: for $\forall d'_i \in [a'_i, d'_i]$:

$$v_i\left(s_i^c[a'_i, d'_i, \theta_{-i}^{\leq d'_i}], A_{-i}^{\leq d'_i}\right) \geq v_i\left(s_i^c[a'_i, d'_i, \theta_{-i}^{\leq d'_i}], A_{-i}^{\leq d'_i}\right)$$

This condition allows the critical signal s_i^c to fall for the early departure d'_i if the later signals are "bad news" for i .

The time-monotonicity conditions ensure that the price i pays is lowest when he reports his interval honestly.

Say that an auction "allocates late" if the winning bidder is never allocated the item until his reported departure; then no bidder would want to misreport a *late* departure since his value at departure for the item would be 0.

THEOREM 1. *The conditions CAD are necessary and sufficient for IC and IR in an online, interdependent value environment, and when the auction allocates late.*

In private values settings $v_i(s) = s_i$ and CAD amount to requiring the existence of a critical value function $v_i^c[a_i, d_i, \theta_{-i}]$ such that $v_i^c[a_i^+, d'_i, \theta_{-i}] \geq v_i^c[a_i, d_i, \theta_{-i}]$ if $[a_i^+, d'_i] \subset [a_i, d_i]$. That recovers Theorem 5 from Hajiaghayi et al. [4].

3.1 Obvious Winner Acceptance

In private value settings, an auction satisfies *consumer sovereignty* (CS) if, with arbitrary fixed values of the other bidders, any bidder can win provided he reports a high enough value. Obvious Winner Acceptance (OWA) makes CS appropriate to our online, interdependent values setting, where we also condition on the item still being available:

DEFINITION 3. *We say that an auction satisfies obvious winner acceptance (OWA) if there is some time \mathbf{T} (the OWA cut) with the following property: whenever some bidder w 's (with $a_w \geq \mathbf{T}$) activity interval is disjoint from any other bidder's activity interval there is some finite value \mathbf{S}_w (that can depend on the other bidders' signals) such that w wins the item with any signal at least as high as \mathbf{S}_w .*

The OWA condition requires that there is some time, past which if w faces no active competition then for some (high enough) signal, bidder w must win. Bidder w is in this case the "obvious winner". If the OWA cut is before bidder w 's $[a_w, d_w]$ interval and the auction is IC then the OWA threshold $\mathbf{S}_w[a_w, d_w, \theta_{-w}]$ must be $s_w^c[a_w, d_w, \theta_{-w}]$.

We call an auction *reasonable* if it satisfies OWA but it does not sell to a bidder reporting his *lowest possible* signal.

By leveraging the IC characterization, one can show that any IC, IR auction satisfying OWA must sell, after the OWA cut, to a bidder whose arrival makes him the only active bidder, *whatever its signal* and regardless of whether some other bidder later arrives during his activity interval:

On each departure d_i , decide as follows if i wins the item:
 Let $s_i^{\geq} = \min\{s'_i : v_i(s'_i, s_{-i}) \geq v_j(s'_i, s_{-i}) \forall j : a_j \leq d_i\}$.
 For all $j \neq i$ such that $d_j \in [d_i - \Delta, d_i]$ let $s_i^{thr}[j]$ be the infimum of all signals s with the following property: whenever i reports some signal $s'_i \geq s$ along with an arrival of d_j , j does not have the highest value across all active or departed bidders (including i) at d_j . Let

$$s_i^* = \max_{j: d_i - \Delta \leq d_j < d_i} \{s_i^{thr}[j]\} \text{ and}$$

$$s_i^c = \max\{s_i^*, s_i^{\geq}\}$$

Sell to i only if $s_i \geq s_i^c$ (s_i^c is i 's critical signal).

Figure 1: IC, IR and reasonable auction for the case of no departure misreports.

THEOREM 2. *There is no reasonable, IC and IR auction in the interdependent values, online environment when bidders can misreport arrival (later only), departure and signal.*

Intuitively, if the only active bidder w at a_w does not win, he may *ex post* regret not reporting a high signal when signals from bidders arriving within $[a_w, d_w]$ raise its value. Imposing IC transforms the OWA requirement of selling to the obvious *winner* into obviously selling to what can be an obvious *loser*, e.g. when w has a very low signal.

3.2 IC auction if departures are known

Even though not possible when bidders are allowed elementary misreports, reasonable IC auctions are possible for more restricted environments. Such an auction for the case of late arrivals and early departures, but no signal misreports, is given in the full version of the paper [9]. We focus here on the more realistic setting of late arrival and signal misreports only (i.e. departures are known): this is the case if, e.g., the seller sets a (possibly different for each bidder) deadline for buying the item.

The auction of Fig. 1 is an IC, reasonable auction in this setting, assuming an upper bound Δ on the patience of any bidder ($d_i - a_i \leq \Delta, \forall i$). The auction will sell to bidder i only at his departure, provided that:

1. at d_i he has the highest value (implied by SCC and $s_i \geq s_i^{\geq}$)
2. no earlier departing bidder j would have had the highest value at d_j , had he observed i 's signal.³ This is a valid scenario if a_i is actually a *late* arrival misreport.

Why does the auction worry about 2.? Say bidder i loses to j whose value depends significantly on i 's signal and great news arrive for i after j 's departure. Then in retrospect (i.e. *ex post*), i may be tempted to stop j from winning by hiding his signal from j via a late arrival misreport.

THEOREM 3. *The auction in Fig. 1 is IC and IR in interdependent, online environments, when v -monotonicity and SCC hold and when bidders can only misreport signals and arrival times but not departure times.*

In the example of Fig. 2: $v_1(s_1, s_{-1}) = \frac{3}{5}s_1 + \frac{2}{5}\max(s_{-1})$, $v_2(s_2, s_{-2}) = \frac{13}{15}s_2 + \frac{2}{15}\text{avg}(s_{-2})$, $v_3(s_3, s_{-3}) = \frac{3}{5}s_3 + \frac{2}{5}400$ and signals $s_1 = 600, s_2 = 690, s_3 = 900$. If 3 reports a_3

³It is enough to check arrivals that coincide with departures since the set of departed bidders only changes on departures.

- at d_1 : $v_1 = 636 < v_2 = 678$.
- at d_2 : $v_1 = 636; v_2 = 698; v_3 = 700$.

If 3 reports $a'_3 = d_3 - \Delta : v_1 = 720; v_2 = 698, v_3 = 700$.
 If we naively sold to a departing bidder if he had the highest value, 3 could do better by reporting a'_3 (and winning) when his true arrival is a_3 (when 1 would win). For the auction in Fig. 1, $s_3^{\geq} = 896.25 = s_3^{thr}[2]$ and $s_3^{thr}[1] = 1000$, since if 3 reports an arrival of d_1 , $v_3 \geq v_1 \iff s_3 \geq 1000$.

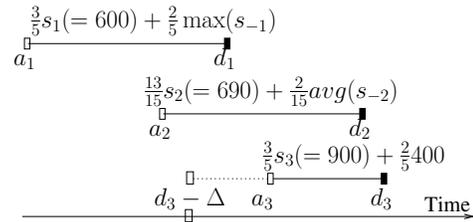


Figure 2: Example setting for the auction in Fig. 1. The auction checks that no earlier arrival of 3 would have resulted in it losing, in case its reported arrival a_3 is actually a late arrival misreport.

The auction in Fig. 1 is reasonable (in particular, it satisfies OWA) under the following mild requirement: $\forall i, \forall$ fixed $s_{-i}, v_i(s_i, s_{-i}) \geq v_j(s_i, s_{-i})$ if s_i is high enough.

When bidders cannot misreport arrivals either, this auction becomes the interdependent second-price auction in [5].

Acknowledgments

This research is supported in part by National Science Foundation grant IIS-0238147.

4. FUTURE WORK

We intend to study optimal auctions (wrt to efficiency and revenue) in our setting. Preliminary investigation shows that naive dynamic programming does not immediately provide IC auctions: additional cross-state constraints must be imposed. Another future research path is to instantiate the contingent-value model as well as more sophisticated models for the dependence of an agent's value on time.

5. REFERENCES

- [1] M. Aoyagi, "Optimal Sales Schemes Against Interdependent Buyers", October 2005. www.iser.osaka-u.ac.jp/library/dp/2005/DP0645.pdf
- [2] P. Dasgupta and E. Maskin, "Efficient Auctions", The Quarterly Journal of Economics. Vol. CXV, May 2000.
- [3] M. Hajiaghayi, R. Kleinberg and D. Parkes, "Adaptive Limited-Supply Online Auctions", ACM EC 2004.
- [4] M. Hajiaghayi, R. Kleinberg and D. Parkes, "Online Auctions with Re-usable Goods", ACM EC 2005.
- [5] V. Krishna, "Auction Theory", Academic Press, 2002.
- [6] T. Ito and D. Parkes, "Instantiating the Contingent Bids Model of Truthful Interdependent Value Auctions", Proceedings of AAMAS 2006.
- [7] A. Juda and D. Parkes, "The Sequential Auction Problem on eBay: An Empirical Analysis and a Solution", Proceedings of ACM EC 2006.
- [8] D. Parkes, "Online Mechanisms", in "Algorithmic Game Theory", edited by N. Nisan, T. Roughgarden, E. Tardos and V. Vazirani, Cambridge Univ. Press, 2007
- [9] F. Constantin, T. Ito and D. Parkes, "Online auctions for bidders with interdependent values" (full version), www.eecs.harvard.edu/~florin/pubs/oabivfull.pdf