Predicting Uncertain Outcomes Using Information Markets*

Yiling Chen Tracy Mullen Chao-Hsien Chu

School of Information Sciences and Technology
The Pennsylvania State University
University Park, 16802
{ychen,tmullen,chu}@ist.psu.edu

Abstract

In this paper, information markets are introduced as a promising mechanism for predicting uncertain outcomes. A model of information markets is proposed. Some fundamental properties on when information markets will converge to the most desirable equilibrium, direct communication equilibrium, are derived.

1 Introduction

Predicting outcomes of related uncertain events is a crucial part of decision making processes. For example, companies rely on forecasts of consumer demand, raw material supply, and possible changes in market regulations, to inform their production decisions. How to accurately estimate outcomes of various random variables that relate to a decision problem is essential to rational decision making.

In this paper, we present a promising tool, *information markets*, for predicting uncertain decision variables. Aiming at gaining deeper understanding of information markets as tools for making predictions, we examine when information markets can make the "best" predictions through rigorous modeling and analysis. By saying "best", we mean that these predictions take advantage of all information across market participants. Thus, compared with other predictions that are made based on less information, they are the best informed predictions. The results of the paper help to understand theoretical properties of information markets, which are currently lack of attention but are in great need for eventually establishing the fundamental underpins of information markets.

2 Background of information markets

An information market can be roughly defined as a financial market that ties to a future event and is specifically designed for forecasting its outcome. The 2004 US presidential vote share market at the Iowa Electronic Markets (IEM) [1] is an example of information markets. To predict who will win the election, the market ties security payoffs to the outcomes of the election. For instance, the security for George W. Bush pays $1 per share if and only if Bush wins the election. Otherwise, it worths nothing. Market participants trade securities based on their expectations about the candidate's winning chances. The security price is hence a group prediction of the probability that George W. Bush will win the election. Since many uncertain future events can be tied with an information market, information markets can function as powerful tools for combining individual opinions and generating an aggregated forecast about the event.

Despite the growing empirical and experimental evidence of the effectiveness of information markets [4], theoretical studies of information markets are relatively rare. Feigenbaum et al. [5] analyzed some computational properties of information markets. They modeled an information market without aggregate uncertainty. Chen et al. [3] studied an information market with aggregate uncertainty. In this paper, we extend the model of Chen et al. [3] by allowing different trader behavior, and investigate the convergence properties of information markets.

3 A model of information markets

There are many possible ways to model information markets, just as many different models for business and financial markets exist. A generic model of information markets should include at least three indispensable components: information structure, market mechanism, and trader behavior. The model introduced below is for a simple class of information markets that trade Boolean securities. It removes the restriction on trader rationality from the model of Chen et al. [3].

3.1 Information structure

Information structure of the market specifies what the state space of the world is, how much information market traders know about the real state of the world, and how information of traders relates to the real state of the world.

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Let $S = \{0,1\}^m$ represents the state space of the world. $s = (s_1, s_2, ..., s_m) \in S$ is a state vector of m dimensions, where s_j can take only one of two state values, 0 or 1, for all $j = 1, ..., m$. Assume there are n traders in the market, where all traders have a common prior probability distribution regarding to state of the world, $\mathcal{P}(s)$: $\{0,1\}^m \to [0,1]$.

The trader's information space is $X = \{0,1\}^n$. Each trader $i = 1, ..., n$ gets a piece of information x_i,which is either 0 or 1, about the state of the world, where $x = (x_1, x_2, ..., x_n) \in X$ is the information vector for all agents. Traders have common knowledge of the probability distribution of x, conditional on the state of the world s, $\mathcal{Q}(x|s)$: $X \times S \to [0,1]$. For example, suppose we have a one dimensional state space (i.e., $s = s_1$). Conditional on $s = 1$ ($s = 0$), the probability to get $x_i = 1$ ($x_i = 0$) is 0.9, and the probability to get $x_i = 0$ ($x_i = 1$) is 0.1. If the trader i gets $x_i = 1$, although he does not know the value of s for certain, he knows that, with probability 0.9, s equals 1.

Our model attempts to capture aggregate uncertainty, which is common to most real world markets. Aggregate uncertainty occurs when even if the information from all traders is pooled together, the state of the world is still not fully determined. In our model, aggregate uncertainty stems from the uncertainty of individual information.

3.2 Market mechanism

Market mechanisms specify what securities are being traded and trading rules of the market. We model our market as predicting the value of a Boolean function $f(s) : \{0,1\}^m \to \{0,1\}$. The value of the function is determined by the true state of the world, which will only be revealed some time in the future. One security is traded in the market, whose payoff is contingent on the value of $f(s)$. Specifically, the security pays off \$1 if $f(s) = 1$ in the future, and \$0 if $f(s) = 0$. The form of f is common knowledge to all traders.

Following Feigenbaum et al. [5], we model the market mechanism as a *Shapley-Shubik market game*with restrictions. The market game proceeds in rounds. Each trader has one unit of the security at the beginning of each round. In each round, each trader puts up the one unit of the security for sell and simultaneously puts up a positive amount of money to buy the security. For simplicity, we assume that there are no restrictions on credit. Then, traders' bids can be represented as a vector $b = (b_1, b_2, ..., b_n)$, where b_i is the amount of money trader i offers to buy securities. The market determines the price of the security by taking the average of all bids in a round, thereby clearing demand and supply. Thus, the price for a round is $p = \frac{\sum_{i=1}^n b_i}{n}$. Only this price p, not individual traders' bids, is publicly announced in each round. All trading occurs at the market price. The market then enters a new round, where each agent has the same initial security holdings as previous rounds. The process continues until an equilibrium is reached, after which prices and bids do not change from round to round.

3.3 Trader behavior

Modeling trader behavior is to model trader's risk preference and rationality, which directly lead to trader's trading strategies. We assume that market traders are risk neutral and myopic. Their utility in each round is the sum of the expected payoff of their security holdings and their money holdings. Thus, for trader i who submits bid b_i in a round, his or her utility is:

$$U_i(b) = \begin{array}{l} Pr_i(f(s)=1) \times (b_i/p + p - b_i) \\ +Pr_i(f(s)=0) \times (p - b_i), \end{array} \tag{1}$$

where $Pr_i(f(s) = 1)$ and $Pr_i(f(s) = 0)$ are trader i's risk-neutral subjective probability assessments about the value of $f(s)$. They usually are assessments conditional on trader i's information. A trader's utility in each round is a function of bids of all traders, because the market clearing price is determined by all submitted bids. Traders are myopic, thus they only care their utility of the current trading round.

We do not put any restriction on trader rationality in the model, because we want our model to capture the most fundamental part of information markets. Different assumptions of trader rationality result in different trading strategies. We construct our analysis in the next section according to different assumptions of trader strategy.

4 Convergence properties of information markets

In this section, we are interested in investigating that under what conditions our information market model converges to an equilibrium where information of traders is fully aggregated. At the equilibrium, security price equals the expectation of the security payoff conditional on all available information, i.e. $E(f(s)|x)$. Such equilibrium is called *direct communication equilibrium* [6] as it can be achieved by market traders through directly communicating their private information with each other. Since the market price incorporates all available information, it is the best prediction that an information market can possibly converge to. The examination is performed respectively under three different assumptions of trader behavior.

4.1 "truth-telling" traders

One of the simplest ways to model trader bidding behavior is to assume that traders bid truthfully, that is, each trader in each round bids his or her current expected payoff of a unit of the security. Expectations are calculated based on probability distribution of the state of the world $\mathcal{P}(s)$, conditional probability distribution of information $\mathcal{Q}(x|s)$, and the information obtained from previously announced market prices. As available information changes when market proceeds, traders revise their expectations accordingly.

Assuming that traders are "truth-telling" seems reasonable when the number of traders in the market is relatively large and complicated strategic reasoning might not effectively improve a trader's utility over simply bidding one's true valuation.

With "truth-telling" traders, our model is the same as that in Chen et al. [3]. We briefly restate the main results on convergence properties of information markets using our notation as the following theorem.

Theorem 1. *(Chen et al.[3]) If the marginal probability distributions of $\mathcal{Q}(x|s)$, $q(x_i|s)$'s, are independent and identical for all $i = 1, ..., n$, and traders expectations of security value are different with different private information, i.e. $Pr(f(s) = 1|xi = 0) \neq Pr(f(s) = 1|xi = 1)$, an information market with "truth-telling" traders converges to direct communication equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

Theorem 1 demonstrates that if the conditional distribution of traders' private information, $\mathcal{Q}(x|s)$, satisfies certain conditions, an information market is guaranteed to converge to the equilibrium that aggregates all information in the market.

4.2 Bayesian traders

When the information market is relatively small, having only a few traders, it is no longer reasonable to assume that traders will truthfully bid their expectations of the security payoff in each round of trading. Thus, we make the assumption that traders are fully rational and bid to maximize their expected utilities in each round. We use the term "Bayesian traders" to represent these expected utility maximizers.

Under this assumption, each trading round of the information market can be viewed as a Bayesian game. We can show that if the game has a Bayesian Nash Equilibrium, Theorem 1 is also valid for information markets with Bayesian traders. We state the results in Corollary 1.1, and provide a sketch of the proof in this section.

Corollary 1.1. *If the following three conditions are met, an information market with Bayesian traders converges to direct communication equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

(a) The marginal probability distributions of $\mathcal{Q}(x|s)$, $q(x_i|s)$'s, are independent and identical for all $i = 1, ..., n$.

(b) Traders expectations of security value are different with different private information, i.e. $Pr(f(s) = 1|xi = 0) \neq Pr(f(s) = 1|xi = 1)$.

(c) The Bayesian Nash Equilibrium exists for the Bayesian game in each round.

Sketch of Proof: A Bayesian trader's expected utility before a trading round is a function of the trader's own information, trader's belief about other traders information, the trader's own bidding strategy, and other traders' bidding strategies. Let $(b_i(0), b_i(1))$ represents a biding strategy for trader i, where $b_i(0)$ represents trader i's bidding strategy when the private information x_i is 0 and $b_i(1)$ represents trader i's bidding strategy when the private information x_i is 1. Thus, the optimal response functions for trader i are obtained by setting the partial first order derivatives of the expected utility with respect to $b_i(0)$ to 0 when x_i is 0, and with respect to $b_i(1)$ to 0 when x_i is 1. The best responses of trader i are functions of bids of all traders. Because $q(x_i|s)$'s are independent and identical for all $i = 1, ..., n$, traders are symmetric, which means that the optimal strategy for all traders should be the same at the Bayesian Nash Equilibrium. This reduces the best response functions to two equations with two variables $b_i^*(0)$ and $b_i^*(1)$. It can then be proved that $b_i^*(0) = b_i^*(1)$ only if $Pr(f(s) = 1|x_i = 0) = Pr(f(s) = 1|x_i = 1)$. In other words, condition (b) in Corollary 1.1 guarantees that $b_i^*(0) \neq b_i^*(1)$. Thus, traders can infer what information the others have from the price of the first round. The information market converges to the direct communication equilibrium at the second round.

The existence of Bayesian Nash equilibrium depends on the prior probability distributions $\mathcal{P}(s)$ and $\mathcal{Q}(x|s)$. Even if it exists, finding the equilibrium strategy is computationally complex.

4.3 Bounded rational Bayesian traders

Fully rational Bayesian traders need to consider other traders' infinite hierarchies of beliefs and form consistent beliefs over them. It is unlikely that traders would be smart enough to consider such a space of infinite beliefs. Thus, we assume *bounded rationality* [7] of traders in this part of analysis.

There are many different ways to model bounded rationality. Without discussing which one is the most appropriate, which is still an open question, we model our bounded rational Bayesian traders as: Each trader forms beliefs about other traders' information and attempts to maximize their expected payoff, but he or she at the same time believes that other traders bid truthfully. This conforms to the understanding of bounded rationality in Bernheim [2]: "We might not expect agents to check the consistency of their beliefs for more than a finite number of levels". In our case, market traders only check the consistency of their beliefs for one level. Information markets with this kind of traders still converge to the direct communication equilibrium when certain conditions are satisfied. We state the result as Corollary 1.2.

Corollary 1.2. *If the following three conditions are met, an information market with bounded rational Bayesian traders converges to direct communication equilibrium within two rounds of trading. At equilibrium, each trader's expectation of the security payoff equals the security price.*

(a) The marginal probability distributions of $\mathcal{Q}(x|s)$, $q(x_i|s)$'s, are independent and identical for all $i = 1, ..., n$.

(b) Traders expectations of security value are different with different private information, i.e. $Pr(f(s) = 1|xi = 0) \neq Pr(f(s) = 1|xi = 1)$.

(c) It is optimal for a trader to bid some positive value.

The proof for Corollary 1.2 is similar to that for Corollary 1.1, but traders face less complicated optimization problems in choosing their bids. Condition (c) in Corollary 1.2 is to ensure that traders have a positive optimal strategy. This depends on the prior probability distributions $\mathcal{P}(s)$ and $\mathcal{Q}(x|s)$ again, but it is less strict than the condition that requires the Bayesian Nash equilibrium exist as in Corollary 1.1.

5 Conclusions

We have introduced information markets as a promising mechanism for predicting uncertain variables that are related to decision making. By examining our model of information markets with three different assumptions of trader strategies, we have proved some fundamental properties on when information markets converge to the direct communication equilibrium, which aggregates all information across traders and is the best possible prediction for information markets. Specifically,

(1) with "truth-telling" traders, sufficient conditions for an information market to converge to the direct communication equilibrium are that (1)distributions of individual traders information conditional on the state of the world are identical and independent, and (2) traders expectations of security value are different with different private information;

(2) with fully rational Bayesian traders, in addition to the conditions in (1), we need that the Bayesian Nash equilibrium exist in each round, to guarantee that the information market converges to the direct communication equilibrium;

(3) with bounded rational Bayesian traders, the existence of positive optimal bid for traders is needed, in addition to the conditions in (1), to guarantee that the information market converges to the direct communication equilibrium.

The existence of Bayesian Nash equilibrium and the existence of positive optimal bid for traders all depend on the prior distributions of the state of the world and the distribution of traders' information conditional on the state of of the world. An implication of our results is that, in order for information markets to aggregate all information and converge to the direct communication equilibrium, they need to be properly designed. Special care is needed to achieve the desired information structure of the market.

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